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$(i,j) - S_C -$ Open set in Bitopological Spaces

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Abstract— The aim of this paper is to introduce a new type of sets in Bitopological spaces called $(i,j) - S_C -$ open sets and give some of their properties. Relationship between this new set and other class of sets are obtained.

Keyword — $(i,j) - S_C -$ open, Semi – open, Regular – open.

1 INTRODUCTION

In 1963 Kelly J. C. [9] was first introduced the concept of bitopological spaces. Where X is a nonempty set and τ_1, τ_2 are topologies on X. In 1963 Levine N. [10] introduced the concept of semi-open sets in topological spaces. Several properties of classic notion have been studied and investigated. In 1971, Crossley and Hildebrand [3] gave some properties of the semi – closure.
In this paper, I introduce the concept of new class of semi – open set in bitopological space, and I find basic properties and relationships with other concept of sets. Throughout this paper, (X, τ_1, τ_2) is a bitopological space and if $A \subseteq X$, then $j - sInt(A)$ and $j - sCl(A)$ denote respectively the semi – interior and semi – closure of A with respect to τ_j of X.

2 PRELIMINARIES

Definition 2.1

A subset A of a space (X, τ) is called:

1. Semi – open [10], if $A \subseteq Cl\big(Int(A)\big)$.
2. Regular open [12], if $A = Int\big(Cl(A)\big)$.

The family of all semi – open (resp., regular open) sets in X is denoted by $SO(X)$ (resp., $RO(X)$). The complement of a semi – open (resp., regular open) set is said to be semi – closed (resp., regular closed) and is denoted by $SC(X)$ (resp., $RC(X)$). The intersection of all semi – closed sets of X containing A is called semi – closure of A and is denoted by $sCl(A)$. The union of all semi – open sets of X contained in A called semi – interior of A and is denoted by $sInt(A)$. A subset A of a space X is called $\delta -$ open [13], if for each $x \in A$, there exists an open set G such that $x \in G \subseteq Int\big(Cl(G)\big) \subseteq A$. A subset A of a space X is called $\theta -$ semi – open [8] (resp., semi$-\theta$ – open [5]), if for each $x \in A$, there exists a semi – open set G such that $x \in G \subseteq Cl(G) \subseteq A$ (resp., $x \in G \subseteq sCl(G) \subseteq A$). The family of all $\delta -$ open (resp., $\theta -$ semi – open, semi$-\theta$ – open) sets in X denoted by $\delta O(X)$ (resp., $\theta SO(X), S\theta O(X)$).A point $x \in X$ is said to be semi $-\theta-$ closure of a subset A of X if $A \cap sCl(U) \neq \emptyset$, for each $U \in SO(X)$ containing x and is denoted by $sCl_\theta(A)$[5]. A subset τ^* of subsets of X is called a supratopology on X [7], if $X, \emptyset \in \tau^*$ and τ^* is closed under arbitrary unions. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $ij -$ clopen [1], if A is i-closed and j-open set in X. A space X is regular if for each $x \in X$ and each open set G containing x, there exist an open set H such that $x \in H \subseteq Cl(H) \subseteq G$ [11].

Definition 2.2

A topological space X is called:

1. $T_1 -$ Space [11] if for every two distinct points of X there exist two open sets each one contains one of the points but not the other.
2. Externally disconnected [5], if $Cl(U) \in \tau$, for every $U \in \tau$.

Theorem 2.3 [4]

Let $(\boldsymbol{X}, \boldsymbol{\tau})$ be a topological space. If $\boldsymbol{G} \in \boldsymbol{\tau}$ and $\boldsymbol{Y} \in \boldsymbol{SO}(\boldsymbol{X})$, then

$\boldsymbol{G} \cap \boldsymbol{Y} \in \boldsymbol{SO}(\boldsymbol{X})$.

Theorem 2.4 [6]

A space $\boldsymbol{X}$ is externally disconnected if and only if $\boldsymbol{RO}(\boldsymbol{X}) = \boldsymbol{RC}(\boldsymbol{X})$.

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Theorem 2.5

For any spaces $\boldsymbol{X}$ and $\boldsymbol{Y}$, if $\boldsymbol{A} \subseteq \boldsymbol{X}$ and $\boldsymbol{B} \subseteq \boldsymbol{Y}$, then

1- $\boldsymbol{sInt_{X\times Y}(A\times B) = sInt_X(A)\times sInt_Y(B)}$. [2].

2- $\boldsymbol{Cl_{X\times Y}(A\times B) = Cl_X(A)\times Cl_Y(B)}$. [11].

Theorem 2.6

Let (Y, τ_Y) be a subspace of a topological space (X, τ), then

1- If $A \in SO(X)$ and $A \subseteq Y$, then $A \in SO(Y)$. [10].

2- If A is closed subset in X and $A \subseteq Y$, then A is closed subset in Y. [11].

3- If $A \in SO(Y)$ and $Y \in SO(X)$, then $A \in SO(X)$. [2].

4- If A is closed in Y and Y is closed in X, then A is closed in X. [11].

3 $(i,j) - S_C -$ OPEN SETS

In this section, I introduce and define a new type of sets in bitopological spaces and I give some of its properties.

Definition 3.1

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(i,j) - S_C -$ open, if A is $j -$ semi – open and for all $x \in A$, there exist an $i -$ closed set F such that $x \in F \subseteq A$. A subset B of X is called $(i,j) - S_C -$ closed if and only if B^C is $(i,j) - S_C -$ open. The family of $(i,j) - S_C -$ open (resp., $(i,j) - S_C -$ closed) subset of X is denoted by $(i,j) - S_CO(X)$ (resp., $(i,j) - S_CC(X)$).

Corollary 3.2

A subset A of a bitopological space (X, τ_1, τ_2) is $(i,j) - S_C -$ open, if A is $j -$ semi – open and it is a union of $i -$ closed sets. This means that $A = \cup F_\gamma$, where A is $j -$ semi – open and F_γ is an $i -$ closed set for each γ.

Corollary 3.3

A subset B of a bitopological space (X, τ_1, τ_2) is $(i,j) - S_C -$ closed, if and only if B is $j -$ semi-closed and it is an intersection of $i -$ closed sets.

Proof. Follows from Corollary 3.2 taking $A = B^C$.

It is clear that every $(i,j) - S_C -$ open subset of a space X is $j -$ semi – open set, but the converse is not true in general, as shown in the following example:

Example 3.4

Let $X = \{a, b, c\}, \tau_1 = \{\emptyset, \{a\}, \{b, c\}, X\}, \tau_2 = \{\emptyset, \{b\}, X\}$, then $(i,j) - S_CO(X) = \{\emptyset, \{b, c\}, X\}$, we see that $\{b\} \in j - SO(X)$, but $\{b\} \notin (i,j) - S_CO(X)$.

It is clear that the union of any family of $(i,j) - S_C -$ open sets in a space X is also $(i,j) - S_C -$ open. The intersection of two $(i,j) - S_C -$open sets is not $(i,j) - S_C -$open set in general, as shown in the following example:

Example 3.5

$X = \{a, b, c, d\}$,

$\tau_1 = \left\{\begin{matrix}\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \\ \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\end{matrix}\right\}$

$\tau_2 = \left\{\begin{matrix}\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \\ \{a, c, d\}, \{b, c, d\}, X\end{matrix}\right\}$

$(i,j) - S_CO(X) = \left\{\begin{matrix}\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \\ \{a, c, d\}, \{b, c, d\}, X\end{matrix}\right\}$

If $A = \{a, d\}$ and $B = \{b, d\}$, then $A \cap B = \{d\} \notin (i,j) - S_CO(X)$.

From the above example I notice that the family of all $(i,j) - S_C -$ open sets is a supratopology and it is not a topology in general.

Lemma 3.6 [11]

A space X is T_1 if and only if every singleton set $\{x\}$ is closed in X.

Proposition 3.7

Let (X, τ_1, τ_2) be a bitopological space. if (X, τ_i) is $T_1 -$ space, then $(i,j) - S_CO(X) = j - SO(X)$.

Proof. Let A be any subset of a space X and $A \in j - SO(X)$. If $A = \emptyset$, then $A \in (i,j) - S_CO(X)$. If $A \neq \emptyset$, let $x \in A$. Since (X, τ_i) is $T_1 -$ space, then by Lemma 3.6 every singleton is $i -$ closed and hence $x \in \{x\} \subseteq A$. Therefore, $A \in (i,j) - S_CO(X)$, hence $j - SO(X) \subseteq (i,j) - S_CO(X)$, but $(i,j) - S_CO(X) \subseteq j - SO(X)$ generally. Thus $(i,j) - S_CO(X) = j - SO(X)$.

Proposition 3.8

A subset A of a bitopological space (X, τ_1, τ_2) is $(i,j) - S_C -$ open set if and only if for each $x \in A$, there exists an $(i,j) - S_C -$ open set B such that $x \in B \subseteq A$.

Proof. Assume that A is $(i,j) - S_C -$ open, then for each $x \in A$, put $A = B$ is $(i,j) - S_C -$ open set containing x such that $x \in B \subseteq A$. Conversely. Suppose that for each $x \in A$, there exist an $(i,j) - S_C -$ open set B such that $x \in B \subseteq A$, thus $A = \cup B_x$, where $B_x \in (i,j) - S_CO(X)$ for each x, therefore A is $(i,j) - S_C -$ open set.

Proposition 3.9

If a subset A of a bitopological space (X, τ_1, τ_2) is ij $-$clopen, then A is $(i,j) - S_C -$ open.

Proof. Since A is $ij -$clopen, so A is $j -$ open and $i -$ closed, this implies that A is $j -$ semi – open and $i -$ closed. Thus for all $x \in A, x \in A \subseteq A$. Hence $A \in (i,j) - S_CO(X)$.

Proposition 3.10

For any bitopological space (X, τ_1, τ_2), if $A \in j - SO(X)$ and $A \in i - S\theta O(X)$, then $A \in (i,j) - S_CO(X)$.

Proof. Let $A \in i - S\theta O(X)$ and $A \in j - SO(X)$. If $A = \emptyset$, $A \in (i,j) - S_CO(X)$. If $A \neq \emptyset$, then for each $x \in A$, there exists an $i -$ semi – open set U such that $x \in U \subseteq i - sCl(U) \subseteq A$, this implies that $x \in i - Cl(U) \subseteq A$, where $i - Cl(U)$ is $i -$ closed and $A \in j - SO(X)$. Therefore, $A \in (i,j) - S_CO(X)$.

Since each $\theta -$ semi – open is semi $- \theta$ – open, so we have the following:

Corollary 3.11

For any subset A of a bitopological space (X, τ_1, τ_2), if $A \in j - SO(X)$ and $A \in i - \theta SO(X)$, then $A \in (i,j) - S_CO(X)$.

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Proposition 3.12

Let (X,τ_1,τ_2) be a bitopological space and $A,B \subseteq X$. If $A \in (i,j)-S_CO(X)$ and B is ij − clopen, then $A \cap B \in (i,j)-S_CO(X)$.

Proof. Since $A \in (i,j)-S_CO(X)$, so $A \in j-SO(X)$ and since B is ij − clopen, then B is j − open, so by Theorem 2.3 $A \cap B \in j-SO(X)$. Let $x \in A \cap B$, then $x \in A$ and $x \in B$ therefore, there exist an i − closed set F such that $x \in F \subseteq A$ and since B is ij − clopen, so B is i − closed set, this implies that $F \cap B$ is i − closed set, therefore,

$x \in F \cap B \subseteq A \cap B$. Thus $A \cap B$ is $(i,j)-S_C-$ open in X.

Proposition 3.13

Let (X,τ_1,τ_2) be a bitopological space and $A,B \subseteq X$.Let (X,τ_i) be an externally disconnected. If $A \in (i,j)-S_CO(X)$ and $B \in j-RO(X) \cap i-RO(X)$, then $A \cap B \in (i,j)-S_CO(X)$.

Proof. Let $A \in (i,j)-S_CO(X)$ and $B \in j-RO(X)$, then $A \in j-SO(X)$ and B is j − open, then by Theorem 2.3 $A \cap B \in j-SO(X)$. Let $x \in A \cap B$, then $x \in A$ and

$x \in B$, therefore there exist an i − closed set F such that $x \in F \subseteq A$. Since (X,τ_i) is extermally disconnected, then by Theorem 2.4 B is i − regular closed set, this implies that $F \cap B$ is i − closed set, therefore $x \in F \cap B \subseteq A \cap B$. Thus $A \cap B$ is $(i,j)-S_C-$ open set in X.

Proposition 3.14

Let X_1, X_2 be two bitopological spaces. If $A \in (i,j)-S_CO(X_1)$ and $B \in (i,j)-S_CO(X_2)$, then $A \times B \in (i,j)-S_CO(X_1 \times X_2)$.

Proof. Let $(x,y) \in A \times B$, then $x \in A$ and $y \in B$. Since $A \in (i,j)-S_CO(X_1)$ and $B \in (i,j)-S_CO(X_2)$, then $A \in j-SO(X_1)$ and $B \in j-SO(X_2)$, and there exist i − closed sets F and E in X_1 and X_2 respectively, such that $x \in F \subseteq A$ and $y \in E \subseteq B$, therefore $(x,y) \in F \times E \subseteq A \times B$. Since $A \in j-SO(X_1)$ and $B \in j-SO(X_2)$, then by Theorem 2.5 (1), $A \times B = j-sInt_X(A) \times j-sInt_Y(B) = j-sInt_{X\times Y}(A \times B)$, so $A \times B \in j-SO(X_1 \times X_2)$. Since F is i − closed in X_1 and E is i − closed in X_2, then by Theorem 2.5 (2), $F \times E = Cl_X(F) \times Cl_Y(E) = Cl_{X\times Y}(F \times E)$. So $F \times E$ is i − closed in $X_1 \times X_2$. Therefore $A \times B \in (i,j)-S_CO(X_1 \times X_2)$.

Corollary 3.15

Let X_1, X_2 be two bitopological spaces. If $A \in (i,j)-S_CC(X_1)$ and $B \in (i,j)-S_CC(X_2)$, then $A \times B \in (i,j)-S_CC(X_1 \times X_2)$.

Proposition 3.16

Let Y be a subspace of a bitopological space (X,τ_1,τ_2). If $A \in (i,j)-S_CO(X)$ and $A \subseteq Y$, then $A \in (i,j)-S_CO(Y)$.

Proof. Let $\mathbf{A} \in (\mathbf{i},\mathbf{j})-\mathbf{S_CO(X)}$, then $\mathbf{A} \in \mathbf{j}-\mathbf{SO(X)}$ and for each $\mathbf{x} \in \mathbf{A}$, there exists an **i** − closed set **F** such that $\mathbf{x} \in \mathbf{F} \subseteq \mathbf{A}$. Since $\mathbf{A} \in \mathbf{j}-\mathbf{SO(X)}$ and $\mathbf{A} \subseteq \mathbf{Y}$, then by Theorem 2.6 (1), $\mathbf{A} \in \mathbf{j}-\mathbf{SO(Y)}$, since **F** is **i** − closed set in **X** and $\mathbf{F} \subseteq \mathbf{Y}$, then by Theorem 2.6 (2), **F** is **i** − closed set in **Y**.
Hence $\mathbf{A} \in (\mathbf{i},\mathbf{j})-\mathbf{S_CO(Y)}$.

Proposition 3.17

Let Y be a subspace of a bitopological space (X,τ_1,τ_2) and $A \subseteq Y$. If $A \in (i,j)-S_CO(Y)$ and $Y \in j-RC(X) \cap i-RC(X)$, then $A \in (i,j)-S_CO(X)$.

Proof. Let $A \in (i,j)-S_CO(Y)$, then $A \in j-SO(Y)$ and for each $x \in A$ there exist an i − closed set F in Y such that $x \in F \subseteq A$. Since $Y \in j-RC(X)$, then $Y \in j-SO(X)$ and since $A \in j-SO(Y)$, so by Theorem 2.6 (3), $A \in j-SO(X)$. Again since $Y \in i-RC(X)$
Then Y is i − closed in X and F is i − closed in Y so by Theorem 2.6 (4), F is i − closed in X. Therefore $A \in (i,j)-S_CO(X)$.

Corollary 3.18

Let Y be a subspace of a bitopological space (X,τ_1,τ_2) and $B \subseteq Y$. If $B \in (i,j)-S_CC(Y)$ and $Y \in j-RO(X) \cap i-RO(X)$, then $B \in (i,j)-S_CC(X)$.

Proposition 3.19

Let (X,τ_1,τ_2) be a bitopological space. if (X,τ_i) is regular space, then $\tau_j \subseteq (i,j)-S_CO(X)$.

Proof. Let $\mathbf{A} \in \boldsymbol{\tau}_\mathbf{j}$. If $\mathbf{A} = \emptyset$, $\mathbf{A} \in (\mathbf{i},\mathbf{j})-\mathbf{S_CO(X)}$. Let $\mathbf{A} \neq \emptyset$, since $(\mathbf{X},\boldsymbol{\tau}_\mathbf{j})$ is regular, so for each $\mathbf{x} \in \mathbf{A} \subseteq \mathbf{X}$, there exists an **i** − open set **G** such that $\mathbf{x} \in \mathbf{G} \subseteq \mathbf{i}-\mathbf{Cl(G)} \subseteq \mathbf{A}$. Thus $\mathbf{x} \in \mathbf{i}-\mathbf{Cl(G)} \subseteq \mathbf{A}$. Since $\mathbf{A} \in \boldsymbol{\tau}_\mathbf{j}$ which implies that $\mathbf{A} \in \mathbf{j}-\mathbf{SO(X)}$, therefore $\mathbf{A} \in (\mathbf{i},\mathbf{j})-\mathbf{S_CO(X)}$. Hence $\boldsymbol{\tau}_\mathbf{j} \subseteq (\mathbf{i},\mathbf{j})-\mathbf{S_CO(X)}$.

4 On $(i,j)-S_C-$ open Operators

Definition 4.1

Let (X,τ_1,τ_2) be a bitopological space and $x \in X$. A subset N of X is said to be $(i,j)-S_C-$ neighborhood of x, if there exists an $(i,j)-S_C-$ open set U in X such that $x \in U \subseteq N$.

Proposition 4.2

In a bitopological space (X,τ_1,τ_2) a subset A of a space X is $(i,j)-S_C-$ open set if and only if it is an $(i,j)-S_C-$ neighborhood of each of its points.

Proof. Obvious.

Proposition 4.3

For any two subsets A, B of a bitopological space (X,τ_1,τ_2) and $A \subseteq B$, if A is $(i,j)-S_C-$ neighborhood of a point $x \in X$, then B is also $(i,j)-S_C-$ neighborhood of the same point x.

Proof. Straightforward.

Definition 4.4

If A is a subset of a bitopological space (X,τ_1,τ_2), then the $(i,j)-S_C-$ interior $\big((i,j)-S_CInt(A)\big)$, the $(i,j)-S_C-$ closure $\big((i,j)-S_CCl(A)\big)$,

1- $(i,j)-S_CInt(A) = \cup\{U: U \subseteq A, U \in (i,j)-S_CO(X)\}$.
2- $(i,j)-S_CCl(A) = \cap\{F: A \subseteq F, X-F \in (i,j)-S_CO(X)\}$.
3- $(i,j)-S_CBd(A) = (i,j)-S_CCl(A)-(i,j)-S_CInt(A)$.

Proposition 4.5

For any subsets A, B of a bitopological space (X,τ_1,τ_2), the following statement are hold:

1- $(i,j)-S_CInt(A)$ is $(i,j)-S_C-$ open set in X.
2- A is $(i,j)-S_C-$ open set if and only if $A = (i,j)-S_CInt(A)$.
3- $(i,j)-S_CInt\big((i,j)-S_CInt(A)\big) = (i,j)-S_CInt(A)$.
4- $(i,j)-S_CInt(\emptyset) = \emptyset$ and $(i,j)-S_CInt(X) = X$.
5- $(i,j)-S_CInt(A) \subseteq A$.
6- If $A \subseteq B$, then $(i,j)-S_CInt(A) \subseteq (i,j)-S_CInt(B)$.
7- $(i,j)-S_CInt(A) \cup (i,j)-S_CInt(B) \subseteq (i,j)-S_CInt(A \cup B)$.
8- $(i,j)-S_CInt(A \cap B) \subseteq (i,j)-S_CInt(A) \cap (i,j)-S_CInt(B)$.

In general the $(i,j)-S_CInt(A) \cup (i,j)-S_CInt(B) \neq (i,j)-S_CInt(A \cup B)$ and $(i,j)-S_CInt(A \cap B) \neq (i,j)-S_CInt(A) \cap (i,j)-S_CInt(B)$, as shown in the following examples:

Example 4.6

Let $X = \{a,b,c,d\}$, $\tau_1 = \{\emptyset,\{b\},\{c\},\{b,c\},X\}$ and $\tau_2 = \{\emptyset,\{a\},\{a,b\},X\}$,
then $(i,j)-S_CO(X) = \{\emptyset,\{a,d\},\{a,c,d\},\{a,b,d\},X\}$

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If $A = \{a,d\}$ and $B = \{c,d\}$, then $(i,j) - S_C\text{Int}(A) = A$, $(i,j) - S_C\text{Int}(B) = \emptyset$ and

$$(i,j) - S_C\text{Int}(A \cup B) = (i,j) - S_C\text{Int}(\{a,c,d\}) = \{a,c,d\} \neq \{a,d\} = (i,j) - S_C\text{Int}(A) \cup (i,j) - S_C\text{Int}(B)$$

Example 4.7
Consider the space X as in Example 3.5. If $A = \{a,d\}$ and $B = \{b,d\}$, then $(i,j) - S_C\text{Int}(A) = \{a,d\}$, $(i,j) - S_C\text{Int}(B) = \{b,d\}$ and $(i,j) - S_C\text{Int}(A \cap B) = (i,j) - S_C\text{Int}(\{d\}) = \emptyset \neq \{d\}$

$$= (i,j) - S_C\text{Int}(A) \cap (i,j) - S_C\text{Int}(B)$$

In general, if $(i,j) - S_C\text{Int}(A) \subseteq (i,j) - S_C\text{Int}(B)$, then is not necessarily that $A \subseteq B$, as shown in the following example:

Example 4.8
Consider the space X as in Example 4.6. if $A = \{c,d\}, B = \{a,d\}$, then$(i,j) - S_C\text{Int}(A) = \emptyset \subseteq (i,j) - S_C\text{Int}(B) = \{a,d\}$, but $A \nsubseteq B$.

Proposition 4.9
If A is any subset of a bitopological space X, then
$(i,j) - S_C\text{Int}(A) \subseteq j - s\text{Int}(A) \subseteq A \subseteq j - s\text{Cl}(A) \subseteq (i,j) - S_C\text{Cl}(A)$.

In general, $(i,j) - S_C\text{Int}(A) \neq j - s\text{Int}(A)$ and $j - s\text{Cl}(A) \neq (i,j) - S_C\text{Cl}(A)$, as shown in the following examples:

Example 4.10
Let $X = \{a,b,c\}, \tau_1 = \{\emptyset, \{a\}, \{b,c\}, X\}$, and
$\tau_2 = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, X\}$, then $(i,j) - S_CO(X) = \{\emptyset, \{a\}, \{b,c\}, X\}$
If $A = \{a,b\}$, then $j - s\text{Int}(A) = \{a,b\} \neq \{a\} = (i,j) - S_C\text{Int}(A)$.

Example 4.11
Let $X = \{a,b,c\}, \tau_1 = \{\emptyset, \{a\}, \{a,b\}, \{a,c\}, X\}$,
$\tau_2 = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, X\}$,then $(i,j) - S_CO(X) = \{\emptyset, \{b\}, \{c\}, \{b,c\}, X\}$.
If $A = \{b\}$, then $(i,j) - S_C\text{Cl}(A) = \{a,b\} \neq j - s\text{Cl}(A) = \{b\}$.

Proposition 4.12
Let A be any subset of a bitopological space X. If $A \cap F \neq \emptyset$ for every $i-$ closed set F of X containing x, then the point x is in the $(i,j) - S_C -$ closure of A.

Proof. Suppose that U be any $(i,j) - S_C -$ open set containing x, then there exist an $i-$ closed set F such that $x \in F \subseteq U$, so by hypothesis$A \cap F \neq \emptyset$, which implies that $A \cap U \neq \emptyset$, for every $(i,j) - S_C -$ open set U containing x. Therefore,
$x \in (i,j) - S_C\text{Cl}(A)$.

Proposition 4.13
For any subsets A, B of a bitopological space X, the following statements are true:

1- $(i,j) - S_C\text{Cl}(A)$ is $(i,j) - S_C -$ closed set in X.
2- A is $(i,j) - S_C -$ closed set if and only if $A = (i,j) - S_C\text{Cl}(A)$.
3- $(i,j) - S_C\text{Cl}\big((i,j) - S_C\text{Cl}(A)\big) = (i,j) - S_C\text{Cl}(A)$.
4- $(i,j) - S_C\text{Cl}(\emptyset) = \emptyset$, and $(i,j) - S_C\text{Cl}(X) = X$.
5- If $A \subseteq B$, then $(i,j) - S_C\text{Cl}(A) \subseteq (i,j) - S_C\text{Cl}(B)$.
6- $(i,j) - S_C\text{Cl}(A) \cup (i,j) - S_C\text{Cl}(B) \subseteq (i,j) - S_C\text{Cl}(A \cup B)$.
7- $(i,j) - S_C\text{Cl}(A \cap B) \subseteq (i,j) - S_C\text{Cl}(A) \cap (i,j) - S_C\text{Cl}(B)$.

Proof. Obvious.

In general, $(i,j) - S_C\text{Cl}(A) \cup (i,j) - S_C\text{Cl}(B) \neq (i,j) - S_C\text{Cl}(A \cup B)$ and $(i,j) - S_C\text{Cl}(A \cap B) \neq (i,j) - S_C\text{Cl}(A) \cap (i,j) - S_C\text{Cl}(B)$, as shown in the following examples:

Example 4.14
Consider the space X as in Example 3.5, if $A = \{a,b\}$ and $B = \{c\}$, then $(i,j) - S_C\text{Cl}(A) = \{a,b\}$, $(i,j) - S_C\text{Cl}(B) = \{c\}$ and

$$(i,j) - S_C\text{Cl}(A \cup B) = (i,j) - S_C\text{Cl}(\{a,b,c\}) = X \neq \{a,b,c\} = (i,j) - S_C\text{Cl}(A) \cup (i,j) - S_C\text{Cl}(B)$$

Example 4.15
Consider the space X as in Example 4.10, if $A = \{a,b\}$ and $B = \{a,c\}$, then $(i,j) - S_C\text{Cl}(A) = X$,$(i,j) - S_C\text{Cl}(B) = X$ and

$$(i,j) - S_C\text{Cl}(A \cap B) = (i,j) - S_C\text{Cl}(\{a\}) = \{a\} \neq X = (i,j) - S_C\text{Cl}(A) \cap (i,j) - S_C\text{Cl}(B)$$

The proof of the following result is obvious.

Proposition 4.16
For any subset A of a bitopological space X, the following statements are true:
$X - [(i,j) - S_C\text{Cl}(A)] = (i,j) - S_C\text{Int}(X - A)$.
$X - [(i,j) - S_C\text{Int}(A)] = (i,j) - S_C\text{Cl}(X - A)$.
$(i,j) - S_C\text{Cl}(A) = X - [(i,j) - S_C\text{Int}(X - A)]$.
$(i,j) - S_C\text{Int}(A) = X - [(i,j) - S_C\text{Cl}(X - A)]$

Proposition 4.17
Let A be a subset of a bitopological space X, then we have:
If $A \in (i,j) - S_CO(X)$, then $j - \text{Cl}_\theta(A) \subseteq (i,j) - S_C\text{Cl}(A)$.
If A is both $(i,j) - S_C -$ open and $(i,j) - S_C -$ closed set, then
$A = (i,j) - S_C\text{Int}\big((i,j) - S_C\text{Cl}(A)\big)$.

Proof. 1) Assume that $x \notin (i,j) - S_C\text{Cl}(A)$, then by Proposition 4.12 there exist an $(i,j) - S_C -$ open U containing x such that $A \cap U = \emptyset$, this implies that
$A \cap (i,j) - S_C\text{Cl}(U) = \emptyset$, since $A \in (i,j) - S_CO(X)$, but $j - s\text{Cl}(U) \subseteq (i,j) - S_C\text{Cl}(U)$ implies that $A \cap j - s\text{Cl}(U) = \emptyset$, therefore $x \notin j - s\text{Cl}_\theta(A)$.
2) If A is both $(i,j) - S_C -$ open and $(i,j) - S_C -$ closed set, then
$(i,j) - S_C\text{Int}\big((i,j) - S_C\text{Cl}(A)\big) = (i,j) - S_C\text{Int}(A) = A$.

Proposition 4.18
For any subset A of a space X, we have the following properties:
$(i,j) - S_C\text{Bd}(A) = (i,j) - S_C\text{Cl}(A) \cap (i,j) - S_C\text{Cl}(X - A)$.
$(i,j) - S_C\text{Bd}(A)$ is $(i,j) - S_C -$ closed set.
$(i,j) - S_C\text{Int}(A) \cap (i,j) - S_C\text{Bd}(A) = \emptyset$.
$(i,j) - S_C\text{Cl}(A) = (i,j) - S_C\text{Int}(A) \cup (i,j) - S_C\text{Bd}(A)$.
$(i,j) - S_C\text{Bd}\big((i,j) - S_C\text{Int}(A)\big) \subseteq (i,j) - S_C\text{Bd}(A)$.
$(i,j) - S_C\text{Bd}\big((i,j) - S_C\text{Cl}(A)\big) \subseteq (i,j) - S_C\text{Bd}(A)$.
$(i,j) - S_C\text{Bd}\big((i,j) - S_C\text{Bd}(A)\big) \subseteq (i,j) - S_C\text{Bd}(A)$.
$(i,j) - S_C\text{Bd}(A) = (i,j) - S_C\text{Bd}(X - A)$.
$(i,j) - S_C\text{Int}(A) = A - [(i,j) - S_C\text{Bd}(A)]$.

Proof. Obvious.

The following result can be proved straightforward statements:

Proposition 4.19
For any subset A of a bitopological space X, we have the following:
If A is both $(i,j) - S_C -$ open and $(i,j) - S_C -$ closed, then $(i,j) - S_C\text{Bd}(A) = \emptyset$.
If A is $(i,j) - S_C -$ closed and $(i,j) - S_C\text{Int}(A) = \emptyset$, then $(i,j) - S_C\text{Bd}(A) = A$.

Denote the $(i,j) - S_C -$ closure of a set A in a subspace Y by $(i,j) - S_C\text{Cl}_Y(A)$.

Proposition 4.20
If A and Y are subsets of a bitopological space X, $A \subseteq Y \subseteq X$ and $Y \in i - RC(X) \cap j - RC(X)$, then $(i,j) - S_C\text{Cl}_Y(A) \subseteq (i,j) - S_C\text{Cl}(A)$.

Proof. Let $x \in (i,j) - S_C\text{Cl}_Y(A)$, then $U \cap A \neq \emptyset$, for each

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$(i,j)-S_C-$ open set U in Y containing x. Since $U \in (i,j)-S_CO(Y)$ and $Y \in i-RC(X) \cap j-RC(X)$, then by Proposition 3.17 $U \in (i,j)-S_CO(X)$, thus $x \in (i,j)-S_CCl(A)$. Therefore $(i,j)-S_CCl_Y(A) \subseteq (i,j)-S_CCl(A)$.

Proposition 4.21

If A and Y are subsets of a bitopological space X, $A \subseteq Y \subseteq X$ and Y is $i-$ clopen and $j-$ clopen, then $(i,j)-S_CCl(A) \cap Y = (i,j)-S_CCl_Y(A)$.

Proof. Let $x \in (i,j)-S_CCl(A) \cap Y$, then $x \in (i,j)-S_CCl(A)$ and $x \in Y$. Take any $V \in (i,j)-S_CO(Y)$ containing x, since Y is $i-$ clopen and $j-$ clopen, this implies that Y is $j-$ regular closed and $i-$ regular closed, then by Proposition 3.17 $V \in (i,j)-S_CO(X)$ containing x and hence $V \cap A \neq \emptyset$, then we get that $x \in (i,j)-S_CCl_Y(A)$. Thus $(i,j)-S_CCl(A) \cap Y \subseteq (i,j)-S_CCl_Y(A)$.

On the other hand, let $x \in (i,j)-S_CCl_Y(A)$, so that $x \in Y$. Let $V \in (i,j)-S_CO(X)$ containing x. Since Y is $i-$ clopen and $j-$ clopen then $V \cap Y \in (i,j)-S_CO(X)$, since $V \cap Y \subseteq Y \subseteq X$, so by Proposition 3.16 $V \cap Y \in (i,j)-S_CO(Y)$. Consequently $A \cap (V \cap Y) \neq \emptyset$, hence $A \cap V \neq \emptyset$, so $x \in (i,j)-S_CCl(X)$, implies that $x \in (i,j)-S_CCl(X) \cap Y$, therefore, $(i,j)-S_CCl_Y(A) \subseteq (i,j)-S_CCl(A) \cap Y$.

Thus, $(i,j)-S_CCl(A) \cap Y = (i,j)-S_CCl_Y(A)$.

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